(with more significant figures) for $n_2 \leqq n_1 \leqq 10$. Other tables are those by FESTINGER[3], by WHITE[4], and by AUBLE[5], which, however, are found to have been criticized for containing more or less serious errors. Using generating functions KEMPERMAN[6] has indicated a method by which to calculate the above probabilities by means of a double-entry table instead of a triple-entry table; he gave such a double-entry table covering the cases for which $u \leqq 40$ and $n \leqq 5$ [$n = \text{Min}(n_1, n_2)$, the size of the smallest sample]. Essentially the same method was quite recently re-discovered (without the use of generating functions) and set forth in a detailed paper by FIX and HODGES[7] who gave the double-entry table needed in case $u \leqq 100$ and $n \leqq 12$. Some old work on partition theory, to be discussed in another paper, gives extensive tables closely connected with the last-mentioned two tables: EULER (1748, 1750) covering the cases $u \leqq 69$ and $n \leqq 11$, and $u \leqq 59$ and $n \leqq 20$, respectively, and MARSANO (1870) covering the case $u \leqq 103$ and $n \leqq 102$.

Now, in some other work on partition theory, BRIOSCHI[8], we found a result, dispensing with tables altogether, which we traduct here in terms of Wilcoxon probabilities. The result is interesting because it answers for the two sample problem a question left open by KRUSKAL[9] with respect to the several sample problem in general: namely, the question of a closed expression for certain probabilities in rank tests for the k ($\geqq 2$) sample problem. Moreover, the result allows one to calculate the exact probabilities in virtually all cases not covered by the tables one may have at hand: in fact, no table is needed at all – though for u large the calculations may turn out to be somewhat cumbersome.

The result, then, is this:

If k is a positive integer as well as i, define

$$\delta(k/i) = 1 \text{ if } k/i \text{ is an integer, and } 0 \text{ if } k/i \text{ is not.}$$

Furthermore define

$$s_k = \sum_{i=1}^{n_2} i \cdot \delta(k/i) - \sum_{i=1}^{n_2} (n_1+i) \cdot \delta(k/(n_1+i)).$$

It is clear that interchanging n_1 and n_2 leaves the value of s_k unaltered; hence if $n_2 < n_1$ the version here given will be chosen; if $n_2 > n_1$ one will prefer to interchange n_1 and n_2 in the expression for s_k. Finally define

$$\Delta(s, 0) = 1, \quad \Delta(s, 1) = s_1,$$

$$\Delta(s, u) = \begin{vmatrix} s_1 & -1 & 0 & \cdots & 0 \\ s_2 & s_1 & -2 & \cdots & 0 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdots & \cdot\cdot \\ s_{u-1} & s_{u-2} & s_{u-3} & \cdots & -(u-1) \\ s_u & s_{u-1} & s_{u-2} & \cdots & s_1 \end{vmatrix} \quad \text{for } u \geqq 2,$$

and

$$s^*_k = 1 + s_k, \quad {}^*s_k = -1 + s_k,$$

while the definition of

$$\Delta(s^*, u) \quad \text{and} \quad \Delta({}^*s, u)$$

be identical with that of $\Delta(s, u)$ except for the symbol s being replaced by s^* and $^*s$, respectively. Then if u be smaller than $\frac{1}{2}n_1 n_2$:

$$P(U_{12} = u; n_1, n_2) = P(U_{21} = u; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta(s, u),$$
$$P(U_{12} \leqq u; n_1, n_2) = P(U_{21} \leqq u; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta(s^*, u),$$

while if in addition $u \geqq 1$:

$$P(U_{12} = u; n_1, n_2) - P(U_{12} = u-1; n_1, n_2) = (n_1!\, n_2!/N!\, u!)\, \Delta({}^*s, u).$$

For example $P(U_{12} \leqq 7; 49, 2) = 0{\cdot}015686$,
$P(U_{12} \leqq 23; 21, 5) = 0{\cdot}028717$.

A self-contained, mainly expository paper with proofs and details on applications is being prepared. Research on other connections between rank tests and the theory of partitions is in progress.

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Zoological Laboratory of the University, Leiden, August 23, 1955.

Zusammenfassung

Gewisse Wahrscheinlichkeiten, welche in der Anwendung von Wilcoxons Test für das Problem der zwei Stichproben wichtig sind, werden mittels einer Determinante geschlossen ausgewertet. Diese Determinante wurde bereits von BRIOSCHI auf ein Problem in der Theorie der Partitionen von ganzen Zahlen angewandt.

from the same source and written by D. WABEKE and C. VAN EEDEN (1955), who give an additional table slightly extending the preceding ones.

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[6] J. H. KEMPERMAN, *De verdelingsfunctie van het aantal inversies in de test van Mann en Whitney*, Rapport T.W. 7, March 1950, Mathem. Centr., Amsterdam.

[7] E. FIX and J. L. HODGES, Jr., Ann. Math. Stat. *26*, 301 (1955).

[8] F. BRIOSCHI, Ann. Sci. mat. fis. (compilati da B. TORTOLINI) *7*, 303 (1856).

[9] W. H. KRUSKAL, Ann. Math. Stat. *23*, 525, 536 (1952).

Electro-optical Shift

Little work appears to have been done on the properties of liquid dielectrics under high electrical fields (direct or alternating). The effect of an electrical field on viscosity[1], on refractive index[2], on internal friction[3] and on the optics[4] of the system has been studied. In conjunction with research on liquid crystals[5] and liquid dielectrics in progress in this laboratory, it was observed that a beam of white light passing through a liquid dielectric can be shifted in its path upon application of an electrical field. Since work, to date, is of a preliminary nature, the present disclosure must be limited in detail.

A typical cell used to study the electro-optical shift consisted of a 50 ml beaker into which two semi-circular metal electrodes are inserted and held rigidly in place. The electrodes in this study were machined from brass. These electrodes are of such a size that they fill the beaker with the exception of a spacing of 5 mm at the center of the beaker and are mounted so that their faces are parallel. The liquid dielectric under study is placed in the space between the electrodes. The spacing between the electrodes is not critical; the important factors controlling it are arcing and voltage supply.

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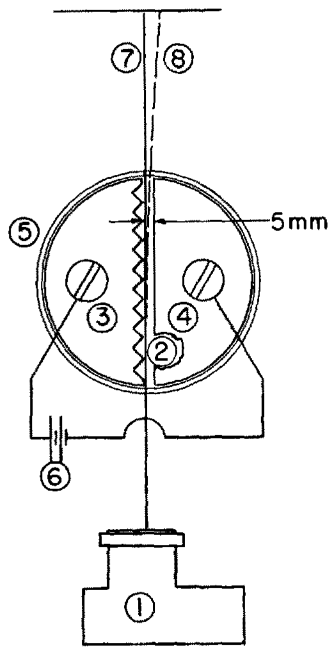
[4] J. KERR, Phil. Mag. *50*, 337 (1875).

[5] *Shedding Light on Liquid Crystals*, Chem. Eng. News *32*, 2962 (1954).

The surfaces of the faces of the electrodes which are in contact with the liquid dielectric should be corrugated for the best results. The tool of a shaper can easily make such corrugations if a çourse feed is used. A satisfactory spacing between ridges on the corrugated surface is one-sixteenth of an inch. Also, other spacings of one-thirty second and one-eighth of an inch between the ridges on the electrical surface have been studied and found to give the shift phenomenon. However, it has been found that a mechanically smooth electrode surface (electrically conducting glass) can also bring about a shift. The degree of shift with the smooth surface is not as pronounced for a given liquid dielectric as with a roughened surface.



Topview of Electro-optical Shift Apparatus. *1* Source of white light producing beam 1 mm in diameter; *2* liquid dielectric in space between electrodes; *3* and *4* electrodes; *5* glass container; *6* 5000 volt source ac or dc; *7* path of light beam in absence of field; *8* path of light beam with field applied.

A beam of white light 1 mm in diameter is passed through the liquid dielectric and along the face of the corrugated electrode. The path of the light is parallel to the electrode surface and perpendicular to the shaper tool marks. It should just contact the ridges of the corrugated surface as it passes along the electrode. Also, the path of the light beam is perpendicular to the direction of the electrical field. The distance of liquid traversed is approximately 4 cm. As far as is known at the present time, this distance is not critical. The strength of the electrical field used across the 5 mm spacing is of the order of 1000 volts per millimeter. The field can be either direct current or alternating current. In addition to the white light, Wratten filters of 425, 525 and 650 millimicrons were used. The light in all three cases was shifted as would be expected.

With the light beam passing along the electrode as described above, the light beam will be shifted from its original path when an electrical field is impressed across the electrodes. A shift of the light beam approximately one degree has been recorded with some liquids. The shift is always away from the electrode surface.

In the preliminary study on the electro-optical shift phenomenon, several organic liquids have been tested in the cell. Typical liquids which have been tested and found to give the shift are: ethyl acetate, absolute ethyl alcohol, absolute methyl alcohol, secondary butyl alcohol, tertiary amyl alcohol, carbon disulfide, nitrobenzene, chloroform, and p-tolyl acetate. It is important that the liquids used should be of a high degree of purity.

Studies are still too fragmentary to state conclusively the cause of this electro-optical shift, but it appears that the shift of the light beam is due to the molecular associations in the liquid dielectric concentrating and orienting in the electrical field of highest gradient thus changing the refractive index of the liquid at the electrode surface.

Research is continuing in our laboratory on the experimental and theoretical interpretation of this phenomenon.

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Zusammenfassung

Erzeugt man in einem flüssigen Dielektrikum, wie wasserfreiem Äthanol, Schwefelkohlenstoff, p-Tolylazetat oder Nitrobenzol, zwischen speziell gestalteten Elektroden ein elektrisches Feld von etwa 10 000 V/cm, so wird ein Lichtstrahl, der die im Feld befindlichen Teile der Flüssigkeit durchsetzt, gekrümmt. Die Ablenkung des Strahls aus der geraden Richtung beträgt in manchen Fällen 1°.

Variations in the Calcification of Cementum and Dentin as Seen by the Use of Microradiographic Technique

Roentgen absorption images of small biological objects were first published by GOBY[1]. He investigated Foraminifera and introduced the name microradiography.

The publication opened new views and possibilities to investigate different tissues on histological basis. DAUVILLIER[2] took a great step forward towards a better technique by using the so-called Lippmann film with its extremely fine grained emulsion.

The method has been further improved by LAMARQUE and TURCHINI[3]. In 1936 SIEWERT[4] among other things showed the possibility of getting enlarged roentgen images working with divergent radiation.

In the nineteen forties, important investigations were made in this subject by CLARK and BOHATYRTSCHUK[5]. The latter e.g. investigated the distribution of vessels. ENGSTRÖM[6] published a quantitative method for the estimations of small amounts of different elements in biological objects in areas down to 10 μ. ENGSTRÖM, ENGFELDT, and AMPRINO[7] have published most interesting microradiograms of bone tissue showing the mineral

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